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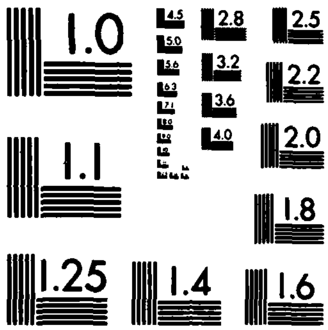

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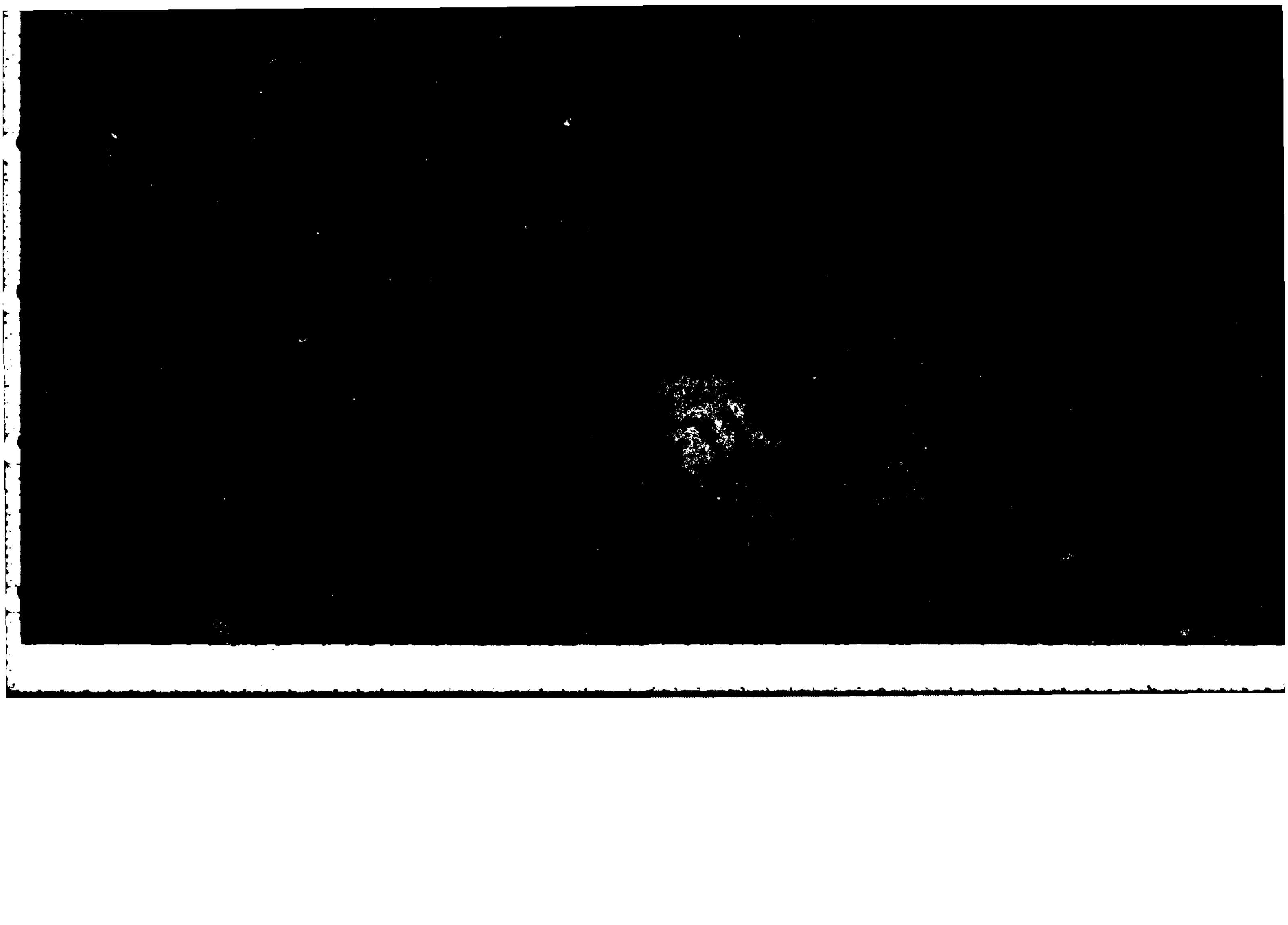

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An M-ary communication channel is described employing orthogonal pulse compression waveforms. An example is presented and analyzed based upon maximum length 7-stage binary shift register (SR) codes (M = 16). Curves of error probability are computed from M-ary communication theory fundamentals and the 7-bit SR code example is shown to provide acceptable error probability (less than 10^{-5}) for output signal-to-noise ratio above 17 dB based upon 120 calculated crosscorrelation functions.

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CONTENTS

ORTHOGONAL PULSE COMPRESSION CODES FOR M-ARY COMMUNICATION

Introduction

Recent work connected with a new system involving pulse compression waveforms has led to a brief study of M-ary communications. This report concerns a straw-man system which is based upon a general form of digital communication referred to by early investigators [1,2] as M-ary communications. The well known frequency shift keying (FSK) system, usually binary, is an example; another even more familiar example is the push button telephone involving 12 "tone" waveforms.

The special class of pulse compression waveforms are identified by a large bandwidth/time (BT) product and by "compressed-pulse" autocorrelation functions. More specifically, these waveforms, when match filtered, deliver an autocorrelation function consisting of a narrow (compressed) pulse shorter than the waveform length by a factor about equal to BT. The compressed pulse is accompanied by time sidelobes extending a time T in both directions from the pulse.

The narrow compressed pulse makes these waveforms suitable where good time resolution is desirable. Moreover, it avoids the need for high peak power associated with short pulses. Another advantage of pulse compression waveforms of interest here is the number of degrees of freedom (large BT) which makes possible synthesis of a number of mutually orthogonal waveforms.

In this report we present an example waveform set defined by eight maximal length shift register codes of length $2^7 - 1 = 127$. Another set of eight is formed by time reversing the above mentioned set. Following brief attention to communication waveform background we present a detailed examination of the 7-bit SR waveform properties. The purpose of the report is to demonstrate, for this set of 16 waveforms, that they are sufficiently orthogonal to provide reliable communication at a rate of four bits per pulse.

Background

Figure 1 illustrates the communication link with which this report is primarily concerned. Pulse compression waveforms are transmitted from a fan beam antenna rotating in azimuth. During the 22 ms the antenna dwells in any direction, a sequence of nearly contiguous pulses is transmitted beginning with a waveform, $s_0(t)$, which we call the framing pulse. Its purpose is to "synchronize" the receiving terminal so that subsequent decisions may be made at points in time at which a matched filter provides the largest response to the corresponding signal. Fixed time delays are ignored for simplicity; e.g., signal $s_i(t)$ representing word w_i is transmitted at t_j and the corresponding compressed pulse from filter, h_i also occurs at t_j.

For example, before s_0 is received the system is in a "ready" state with the constant false alarm rate (CFAR) detector continuously comparing each digitized sample of the filtered signal, Z_0, with a threshold

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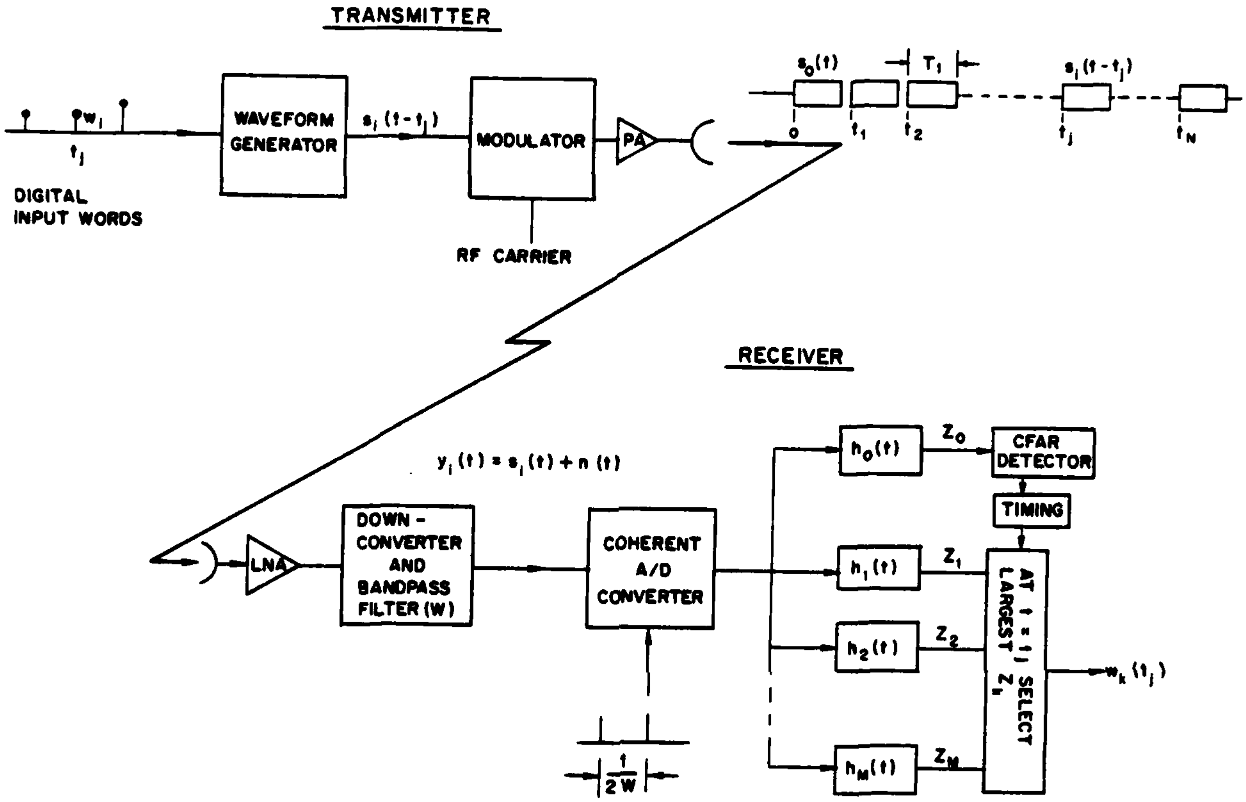


Fig. 1 — M-ary communication link block diagram

computed from eight preceding and eight succeeding samples.* Since s_0 is received first, and since time intervals to succeeding pulses are known, the narrow (compressed) output, to provide estimates, $\hat{t}_j$, of the times, t_j at which subsequent pulses will be received. From these estimates the timing unit enables the comparator to choose the filter producing the largest response; i.e., if Z_k is the largest response the receiver concludes that w_k was transmitted at time $\hat{t}_j$. Thus if $k \neq i$ the system is in error and we are interested here in predicting signal levels needed to maintain the occurrence of errors at adequately low rates. Note that the index i identifies the signal actually transmitted at t_j.

It is important to note that, for an M-ary communication link of the type illustrated in Fig. 1, the receiver (M matched filters) is optimum where interference is due to white, Gaussian noise. Specifically, it is optimum in the sense that the likelihood ratio is maximized. Proofs of this assertion along with needed mathematical support are found in [1] and [2]. These references also show that an optimum estimate of the time of occurrence of a waveform (in this case, s_0) is obtained by selecting the largest output from a matched filter (in this case, h_0).

Waveform Selection

The preceding section states that the matched filter receiver of Fig. 1 is optimum in the sense of maximum likelihood detection and minimum error in estimating t_j, given the transmissions sketched in the figure. Here we discuss factors influencing the selection of pulse compression waveforms.

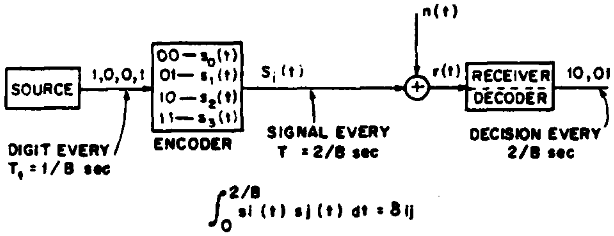
If the object is simply the transmission of digital numbers at a speed determined by system requirements we are faced with the classical choice discussed by Shannon [4] and other information theorists. Suppose binary digits recurring every T_1 seconds (Fig. 1) are to be communicated. How long should the waveform, $s_i(t)$ be and how many are needed? Logical alternatives are listed below:

1. One of two orthogonal waveforms represent binary zero and one every T_1 seconds.

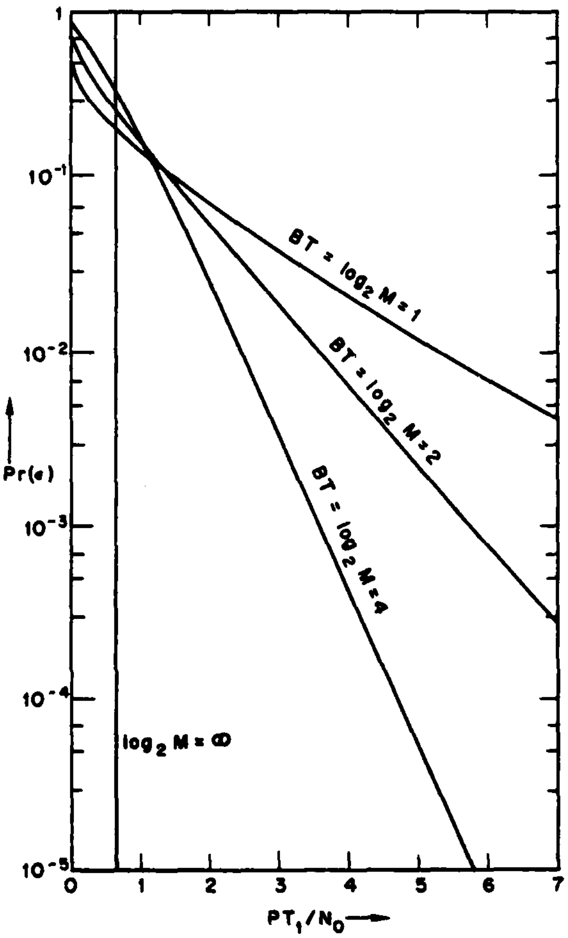
2. One of four orthogonal signals represent 00, 01, 10, or 11 every $2T_1$ seconds (See Fig. 2-a).

3. In general, one of M orthogonal signals represent one of the M combinations of digits which may be formed from a binary sequence of length, $T_1 \log_2 M$ seconds.

* A variety of adaptive thresholding designs have appeared in the literature. Mitchell and Watkins [3] show that if the threshold is proportional to the weighted sum of 16 samples (not including the test sample) the loss is between 1 and 2 dB. This assumes probability of detection, $P_d = 0.9$ and probability of false alarm, $P_{fa} = 10^{-6}$.



(a) M = 4 communication system example
(B = width of spectra of S_i (t))



(b) — Probability of decision error: M orthogonal signals, each of power P in presence of noise of power density, N_0.

Fig. 2 — Effect of changing M on M-ary Communication System (a) M = 4 example; (b) graph of error probability $P_r(\epsilon)$ vs M and signal-to-noise ratio (PT/N_0)

Note that, regardless of M, the system provides communication at a rate of one binary digit every T_1 seconds. Thus it is natural to look for a value of M which maximizes efficiency; i.e., how does M influence error rate for a given signal-to-noise ratio (SNR)? Figure 2-b contains a plot of error probability vs PT_1/N_0 and M where P is the power of the rectangular waveform and N_0 is the noise power density. Clearly from the figure, M should be large. Parameters $T_1 \approx 1/B$ and $N_0 = kT_s$ (where B is the receiver bandwidth, k is Boltzman's constant, and T_s is the receiver noise temperature) are system constants; thus the figure shows that less signal energy is needed as longer waveforms are employed (large M) while maintaining information rate constant.

For the purposes of this report we have selected a set of waveforms derived from 7-stage shift registers. Two examples are specified in Table I with block diagrams illustrating their synthesis. Table II taken from [5] identifies 7-bit and 8-bit maximum length binary codes along with the largest values of their autocorrelation function sidelobes.

Properties of the 7-bit Maximal Length Shift Register Codes

In order to send a four-bit word on each pulse its waveform must be selected from a set of 16. This section presents calculated auto- and crosscorrelation functions for 16 of the 18 7-bit, maximal length SR codes. Note that for M = 16, 17 waveforms are needed including $s_0(t)$ (See Fig. 1). Here we assume that a 17th waveform will have properties similar to the 16 discussed in the following paragraphs.

The 7-bit SR codes of Table II are plotted with their autocorrelation functions in Fig. 3. The numbers separated by commas identify the SR taps which are modulo-two summed and fed back to the SR input to generate each code. The first two codes on the left correspond to the codes of Table I. As shown in Table II, the worst (highest) sidelobe of the set is 22 dB below the autocorrelation peak, Figure 3 represents all 16 waveforms of the set since time reversal has no effect upon the autocorrelation function as indicated below in Eq. (1). This is shown in general for any signal, $s_i(t)$ of length, T in Eq. (1) below.

$$R_i(\tau) = \int_0^T s_i(t)s_i(t+\tau)dt$$
$$= \int_0^T s_i(T-t)s_i(T-t+\tau)dt \qquad (1)$$

It is clear from Figs. 1 and 3 that large sidelobes of $R_0(\tau)$ could cause errors in synchronization, hence a total loss of the message. However, since the sidelobes shown in Fig. 3 are more than 22 dB below the peak (and tend to exhibit a uniform amplitude distribution), we assume that the CFAR detector will ignore sidelobe peaks; the CFAR threshold setting is based upon observations of 8 samples ahead and 8 samples behind the test sample. Finally, we assume that the signal power in waveform, $s_0(t)$ will be set so that probabilities of detection and false alarm (P_d and P_{fa} respectively) are acceptable. For example, if the signal-to-noise ratio, SNR = 17 dB then if $P_{fa} = 10^{-6}$, $P_d = 0.99999$ (non-fluctuating path) [6].

Table I - Binary Code Examples

(7-bit)

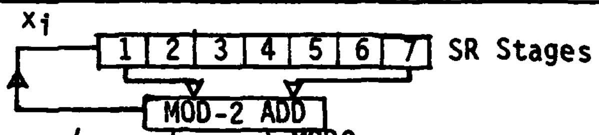
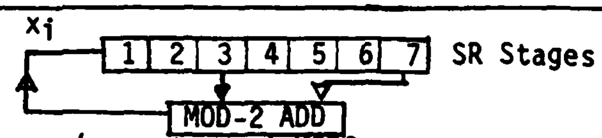
Polynomial Octal	Polynomial Binary	Block Diagram
203	010000011	 $x_i = (x_{i-1} + x_{i-7})$ MOD2
211	010001001	 $x_i = (x_{i-3} + x_{i-7})$ MOD2

Table II - 7-Bit and 8-Bit Maximum-length Binary Codes

Degree	Polynomial (Octal)	Largest Autocorrelation Sidelobe Amplitude
7 (127)	203	- 9 (24 dB)*
	211	- 9
	235	- 9
	247	- 9
	253	-10 (22 dB)*
	277	-10
	313	- 9
	357	- 9
8 (255)	435	-13 (26 dB)*
	453	-14 (25 dB)*
	455	-14
	515	-14
	537	-13
	543	-14
	607	-14
	717	-14

* Below peak autocorrelation equal to 127

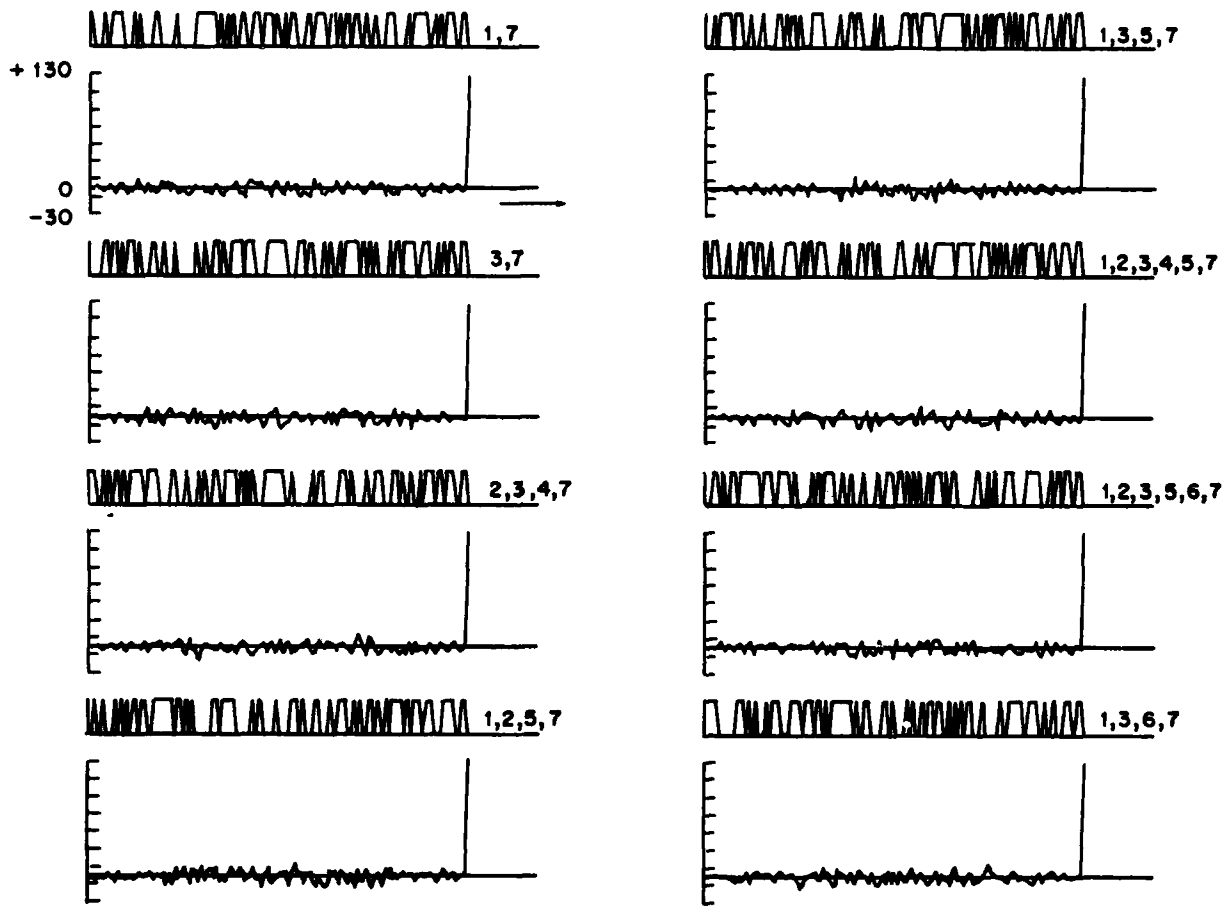


Fig. 3 — Seven stage maximum-length shift register codes and autocorrelation functions

Optimum Code Length

Van Trees [3] presented a fundamental curve of M-ary communications based on the availability of M mutually orthogonal signals of length $T_1 \log_2 M$ which has been reproduced as Fig. 2-b. Note that, for a given SNR, $P_r(\varepsilon)$ (error probability) may be reduced to arbitrarily small values by making M sufficiently large. While this is an important property of M-ary communications it depends upon mathematical orthogonality. Thus, optimum code length for real signals must be based upon error probability data of the type in Fig. 2-b but where non-zero auto- and crosscorrelation values are accounted for.

Figure 4 contains graphs of crosscorrelation functions selected from the set of 120 functions implied by Eq. (2) for the 16 waveforms defined earlier.

$$R_{ij}(\tau) = \int_0^T s_i(t)s_j(t + \tau)dt, \quad i \neq j \qquad (2)$$

where for $1 \leq i,j \leq 8$, the s_i are given in Table II (7-bit case) and Fig. 3. For $9 \leq i,j \leq 16$, $s_i(t) = s_{i-8}(T - t)$ where $T = 127/B$. All of these cases have been plotted ($R_{ij}(\tau) = R_{i+8,j+8}$) and none have values higher than 35 (11 dB below 127, the autocorrelation peak).

An important property is readily apparent from Fig. 4. The highest sidelobe levels (SLL) appear at values of relative delay near zero where the timing unit (Fig. 1) will sample all filter outputs to determine which of the 16 waveforms, $s_i(t)$ was sent. Another important factor appears in Table II where SLL values are relatively lower for the longer code, again, larger values of M lead to reduced probability of error.

To determine the minimum waveform length for a given M and SLL would require results currently unavailable to the best of the writer's knowledge. Current and advanced technology M-ary systems do not normally make use of the time coordinate for range measurement and thus may use time modulation requiring only one matched filter. Moreover, M-ary frequency shift keying (FSK) is often used. However, the system of interest here requires wide (megahertz) bandwidth so that the bandwidth of a 16-ary FSK receiver would be wider by a factor of 16. While orthogonality is compromised (all 16 signals share the same frequency band) we show in this report that auto- and crosscorrelation SLL's of simple 7-stage SR codes are more than adequate for a four-bits-per-pulse (M = 16) system.

Thus, by optimum code length we mean the minimum length providing adequate SLL for a four-bits-per-pulse system. It is known [7] that the number of linear maximum length (ML) SR codes decreases with length according to the following expression. Number of ML Codes = $\phi(2^n-1)/n$, where $\phi(i)$ is the number of integers, including 1, which are less than i and have no factors other than 1 in common with i. Since only six 6 bit maximum length (ML) codes exist we must use at least 7 stages to implement a 4-bit word; there are 18 ML 7-stage codes.

The next section of the report shows that this set is adequate to provide four bits of information per pulse with acceptable error rate.

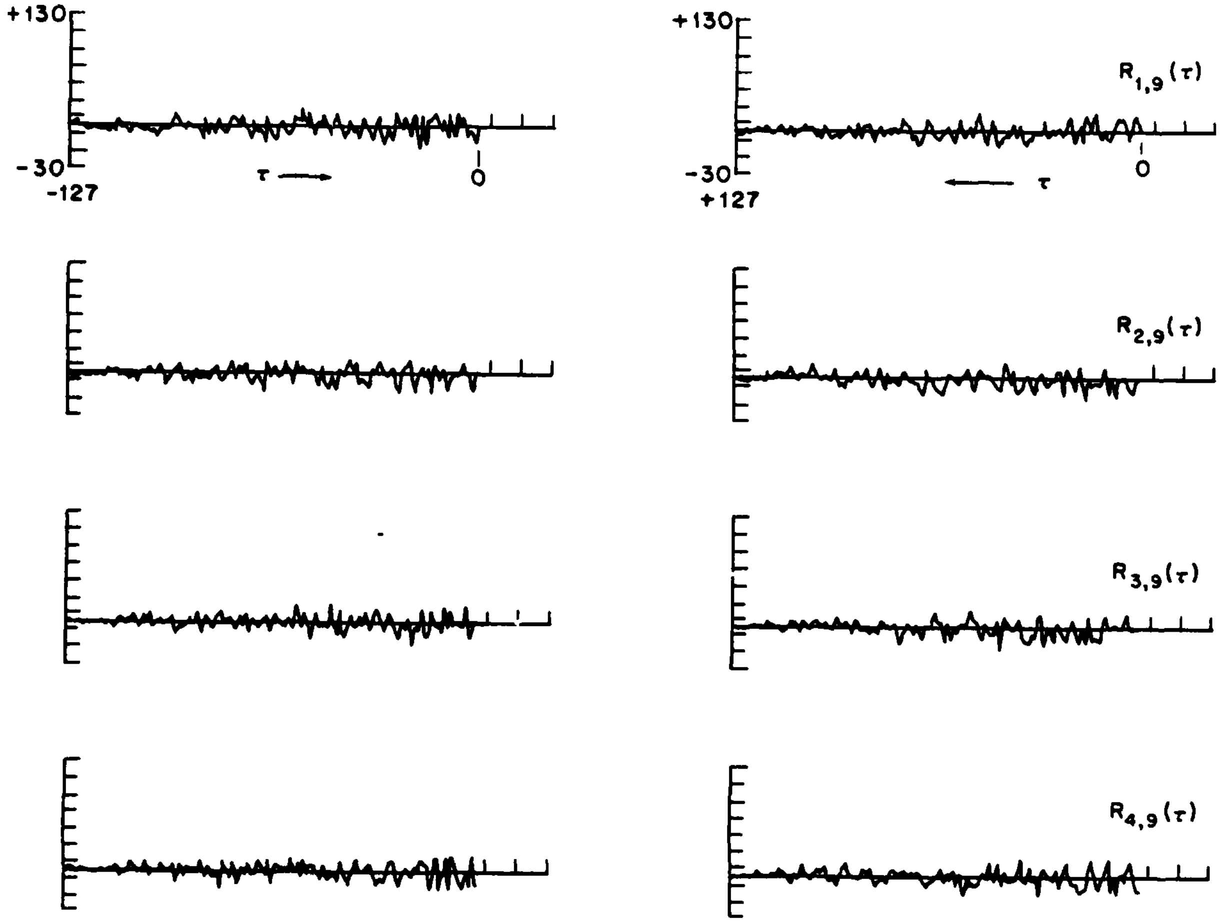


Fig. 4 — Sample from 120 crosscorrelations of 16 7-bit SR codes

On the Effect of Non-zero Crosscorrelation on Error Probability

The receiver of an M-ary communication system must contain processing represented in Fig. 1 by an M-ary comparison of the response magnitudes from M matched filters. If the i^{th} signal is sent, the i^{th} matched filter output is proportional to $R_i(\tau)$. Any other filter, j will provide at its output, $R_{ij}(\tau)$. The purpose of the framing pulse, CFAR detector, and timing unit is to control times at which this comparison is made. Thus, if M = 16 the largest among $|R_i(\delta) + n_i(\delta)|$ and 15 corresponding values of $|R_{ij}(\delta) + n_j(\delta)|$ are compared where δ is the error in the timing estimate (synchronization error) and $n_i(t)$ is the noise at the i^{th} matched filter output.

If the $s_i(t)$ are orthogonal* and if $\delta = 0$, then the error probability $P_i(\varepsilon)$, assuming $n_i(t)$ derives from white Gaussian noise, is well known and is plotted for various M as a function of signal-to-noise-ratio in Fig. 5. $P_r(\varepsilon)$ is defined to be the probability that $|R_i(0) + n_i(0)| < \max n_j(0)$, $i \neq j$.

Since the timing error $\delta \neq 0$ and since $R_{ij}(0) \neq 0$ we must re-calculate $P_r(\varepsilon)$ to equal the probability that $|R_i(\delta) + n_i(\delta)| < \max |R_{ij}(\delta) + n_j(\delta)|$, $i \neq j$. While this problem has been studied [3], a detailed examination is outside the scope of this report. Here we make the simplifying assumption that the $R_{ij}(\delta)$ is a random variable which is independent of the n_j. This makes possible the use of the results plotted in Fig. 5 (from Fig. 4.23, Ref. (3)) by the argument following.

The SNR at the output of a matched filter is known to be $E/N_0 = S/N$ where S and N are defined to be the peak signal and noise powers at the output of a filter containing the corresponding signal at its input. E and N_0 were previously defined to be the signal energy and noise power density respectively, the same for all signals and all filters. By the previous assumption, the effective "noise" power from each of the M-1 filters containing $R_{ij}(\delta) + n_j(\delta)$ will be regarded as

$$N_e = k_{ij}S + N \tag{4}$$

where $k_{ij} = R_{ij}/S$, $i \neq j$, the crosscorrelation-to-peak signal power ratio.

Secondly, we assume that the curves of Fig. 5 may be applied by defining an effective SNR, $\mathcal{R}_e = S/N_e$ to account for the effect of non-zero crosscorrelations. From this definition and Eq. (4) we may write

$$\mathcal{R}_e = \frac{\mathcal{R}}{1 + k_{ij}\mathcal{R}} \tag{5}$$

where $\mathcal{R} = S/N$.

The accuracy of this method could be questioned for small $\mathcal{R}$ and M since it violates the "equal noise power" assumption governing calculations plotted in Fig. 5. That is, in practice the noise power in the "matched"

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* $R_{ij}(0) = 0$, $i \neq j$.

channel is due only to $n_i(t)$; making use of the curve carries the implicit assumption that N_e is the noise power in all channels. For large values of M and R of interest here the change in error probability is expected to be negligible, especially since noise adds to and subtracts from the peak signal level, $R_i(\delta)$.

Equation (5) is plotted in Fig. 6 for several values of R and for k_{ij} from -30 dB to 0 dB which spans the range typical of phase codes from small BT up to about 10^3. Referring back to Fig. 5 we note that if R = 20 then $P_r(e) \simeq 10^{-5}$ for M = 16. A 3 dB increase in SNR lowers $P_r(\varepsilon)$ about two orders of magnitude.

In Fig. 6 we note that for k_{ij} = -20 dB, an input SNR, R = 30 = 15 dB is required to achieve R_e = 14 dB and $P_r(\varepsilon) = 10^{-5}$; for R = 16 dB, R_e = 15 dB and $P_r(\varepsilon) = 10^{-7}$. For k_{ij} = -10 dB we note that even if R = 50 = 17 dB, R_e = 10 dB with $P_r(\varepsilon) = 10^{-2}$; further increases in R have little effect upon error probability. While only the 7-bit SR codes are presented here more complex codes may exist (e.g., Gold codes) with somewhat better SLL. We may conclude from the foregoing, however that a system based upon the 16 7-bit codes described earlier may be implemented with adequately small error.

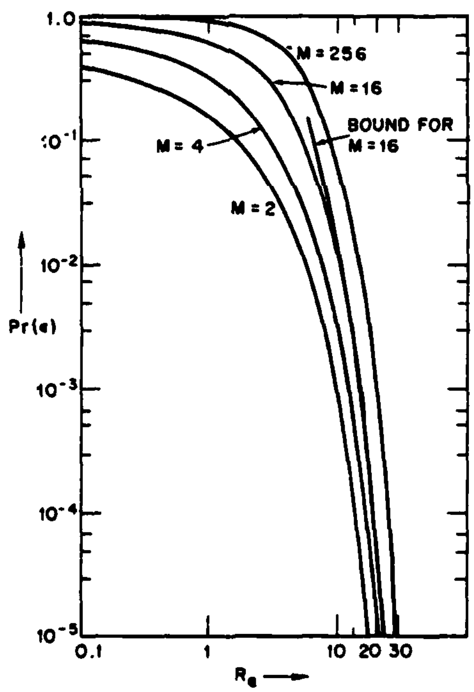

Fig. 5 — Error probability R_1 for M orthogonal signals vs effective signal-to-noise ratio (R_e).

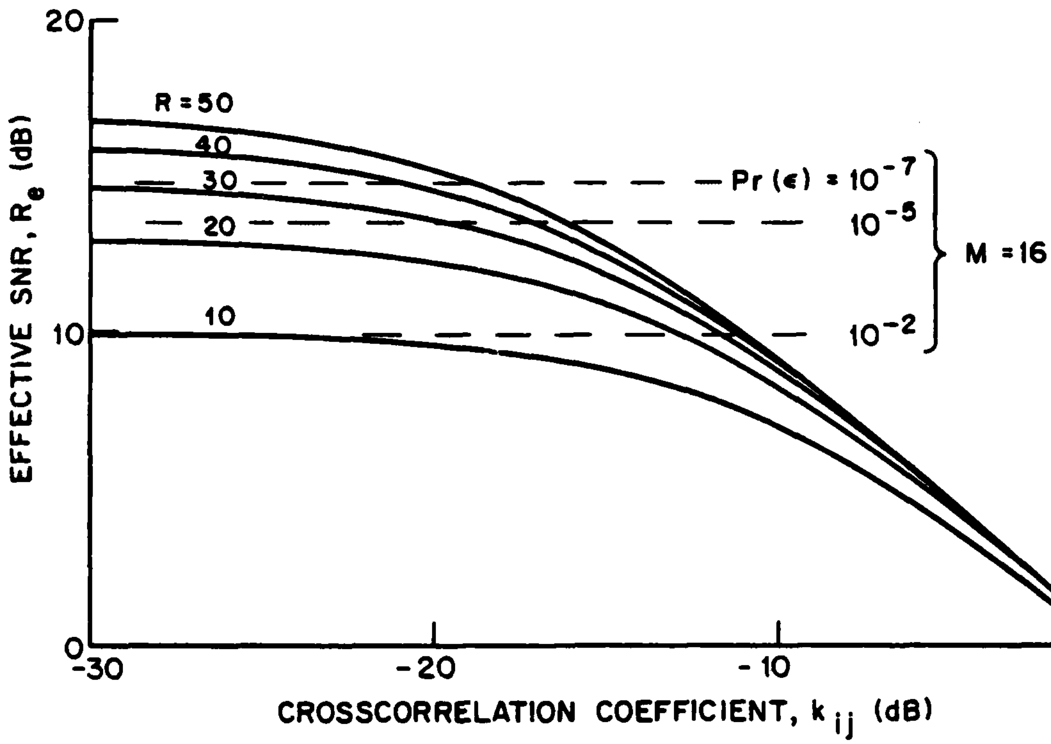

Fig. 6 — Influence of crosscorrelation levels on M-ary communications

Conclusions

We have examined fundamental design trade-offs affecting the reliability of an M-ary communication system based on large time-bandwidth pulse compression signals occupying the same band in frequency space. A four-bit (M = 16) example was analyzed and demonstrated to provide acceptable error rates including the effects of -15 dB cross-correlation to peak auto-correlation ratio ($P_r(\varepsilon) = 10^{-5}$ for SNR = 17 dB). This is considered a worst case and further study of the effect of code starting phases on error probability is recommended.

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